

# The angular size of the Cepheid $\ell$ Car: a comparison of the interferometric and surface brightness techniques

Pierre Kervella<sup>1,7</sup>, Pascal Fouqué<sup>1,2</sup>, Jesper Storm<sup>3</sup>, Wolfgang P. Gieren<sup>4</sup>, David Bersier<sup>5</sup>,  
Denis Mourard<sup>6</sup>, Nicolas Nardetto<sup>6</sup>, Vincent Coudé du Foresto<sup>7</sup>

pkervell@eso.org, pfouque@eso.org, jstorm@aip.de, wgieren@coma.cfm.udec.cl,  
bersier@stsci.edu, Denis.Mourard@obs-azur.fr, Nicolas.Nardetto@obs-azur.fr,  
vincent.forest@obspm.fr

## ABSTRACT

Recent interferometric observations of the brightest and angularly largest classical Cepheid,  $\ell$  Carinae, with ESO's VLT Interferometer (VLTI) have resolved with high precision the variation of its angular diameter with phase. We compare the measured angular diameter curve to the one we derive by an application of the Baade-Wesselink type infrared surface brightness technique, and find a near-perfect agreement between the two curves. The mean angular diameters of  $\ell$  Car from the two techniques agree very well within their total error bars (1.5%), as do the derived distances (4%). This result is an indication that the calibration of the surface brightness relations used in the distance determination of far away Cepheids is not affected by large biases.

*Subject headings:* Stars: distances – Stars: fundamental parameters – Stars: variables: Cepheids – Stars: oscillations – Techniques: interferometry – extragalactic distance scale

## 1. Introduction

Cepheid variables are fundamental objects for the calibration of the extragalactic distance scale. Distances of Cepheids can be derived in at least two different ways: by using their observed mean magnitudes and periods together with a period-luminosity relation, or by applying a Baade-Wesselink (hereafter BW) type technique to determine their distances and mean diameters from

their observed variations in magnitude, color and radial velocity. This latter technique has been dramatically improved by the introduction of the near-infrared surface brightness method (hereafter IRSB) by Welch (1994), and later by Fouqué & Gieren (1997) who calibrated the relation between the  $V$ -band surface brightness and near-infrared colors of Cepheids. For this purpose, they used the observed interferometric angular diameters of a number of giants and supergiants bracketing the Cepheid color range. This method has been applied to a large number of Galactic Cepheid variables, for instance by Gieren et al. (1997), Gieren et al. (1998), and Storm et al. (2004).

Applying the surface-brightness relation derived from stable stars to Cepheids implicitly assumes that the relation also applies to pulsating stars. The validity of this assumption can now be addressed by comparing direct interferometric measurements of the angular diameter variation of a Cepheid to the one derived from the IRSB technique. It has recently been shown by Kervella et al. (2004), hereafter K04, that the VLT In-

<sup>1</sup>European Southern Observatory, Casilla 19001, Santiago 19, Chile

<sup>2</sup>Observatoire Midi-Pyrénées, UMR 5572, 14, avenue Edouard Belin, F-31400 Toulouse, France

<sup>3</sup>Astrophysikalisches Institut Potsdam, An der Sternwarte 16, D-14482 Potsdam, Germany

<sup>4</sup>Universidad de Concepción, Departamento de Física, Casilla 160-C, Concepción, Chile

<sup>5</sup>Space Telescope Science Institute, 3700 San Martin Drive, Baltimore, MD 21218, USA

<sup>6</sup>GEMINI, UMR 6203, Observatoire de la Côte d'Azur, Avenue Copernic, F-06130 Grasse, France

<sup>7</sup>LESIA, Observatoire de Paris-Meudon, 5, place Jules Janssen, F-92195 Meudon Cedex, France

terferometer on Paranal is now in a condition to not only determine accurate *mean* angular diameters of nearby Cepheid variables, but to follow their angular diameter *variations* with high precision. Using the Palomar Testbed Interferometer, Lane et al. (2000, 2002) resolved the pulsation of the Cepheids  $\zeta$  Gem and  $\eta$  Aql as early as 2000, but the comparison we present in this letter is the first where error bars on the derived distance and linear diameter are directly comparable at a few percent level between the interferometric and IRSB techniques.

The star that we will discuss in this letter,  $\ell$  Car, is the brightest Cepheid in the sky. Its long period of about 35.5 days implies a large mean diameter, which together with its relatively short distance makes it an ideal target for resolving its angular diameter variations with high accuracy. In this paper, we will compare the interferometrically determined angular diameter curve of  $\ell$  Car with that determined from the IRSB technique, and we will demonstrate that the two sets of angular diameters are in excellent agreement. Based on the available high-precision angular diameter and radial velocity curves for this star, we will also derive a revised value of its distance and mean radius.

Several authors (Sasselov & Karovska 1994; Marengo et al. 2003, 2004) have pointed out potential sources of systematic uncertainties in the determination of Cepheid distances using the interferometric BW method. In particular, imperfections in the numerical modeling of Cepheid atmospheres could lead to biased estimates of the limb darkening and projection factor. We will discuss the magnitude of these uncertainties in the case of  $\ell$  Car.

## 2. Interferometric observations

The interferometric observations of  $\ell$  Car were obtained with the VLT Interferometer (Glademmann et al. 2000), using its commissioning instrument VINCI (Kervella et al. 2000, 2003) and 0.35 m test siderostats. This instrument recombines the light from two telescopes in the infrared  $K$  band (2.0–2.4  $\mu\text{m}$ ), at an effective wavelength of 2.18  $\mu\text{m}$ . A detailed description of the interferometric data recorded on  $\ell$  Car can be found in K04.

The limb darkening (LD) models used to de-

rive the photospheric diameters from the fringe visibilities were taken from Claret (2000). The correction introduced on the uniform disk (UD) interferometric measurements by the limb darkening is small in the  $K$  band: for  $\ell$  Car, we determine  $k = \theta_{\text{UD}}/\theta_{\text{LD}} = 0.966$ . Considering the magnitude of this correction, a total systematic uncertainty of  $\pm 1\%$  appears reasonable. However, until the limb darkening of a sample of Cepheids has been measured directly by interferometry, this value relies exclusively on numerical models of the atmosphere. This is expected to be achieved in the next years using for instance the longest baselines of the VLTI (up to 202 m) and the shorter  $J$  and  $H$  infrared bands accessible with the AMBER instrument (Petrov et al. 2000).

The LD correction is changing slightly over the pulsation of the star, due to the change in effective temperature, but Marengo et al. (2003) have estimated the amplitude of this variation to less than 0.3% peak to peak in the  $H$  band (for the 10 days period Cepheid  $\zeta$  Gem). It is even lower in the  $K$  band, and averages out in terms of rms dispersion. As a consequence, we have neglected this variation in the present study.

The limb-darkened angular diameter measurements are listed in Table 1. Two error bars are given for each point, corresponding respectively to the statistical uncertainty (internal error) and to the systematic error introduced by the uncertainties on the assumed angular diameters of the calibrator stars (external error). The phases given in Table 1 are based on the new ephemeris derived in Sect. 4. These measurements were obtained during the commissioning of the VLTI, and part of them are affected by relatively large uncertainties (3–5%) due to instrumental problems. However, the precision reached by VINCI and the test siderostats on this baseline is of the order of 1% on the angular diameter, as demonstrated around the maximum diameter phase.

## 3. The infrared surface brightness technique

The IRSB technique has been presented and discussed in detail in Fouqué & Gieren (1997) (hereafter FG97). In brief, the angular diameter curve of a given Cepheid variable is derived from its  $V$  light and  $(V - K)$  color curve, appropri-

ately corrected for extinction. It is then combined with its linear displacement, which is essentially the integral of the radial velocity curve. A linear regression of pairs of angular diameters and linear displacements, obtained at the same pulsation phases, yields both the distance, and the mean radius of the star.

While there are several sources of systematic uncertainty in the method, as discussed in Gieren et al. (1997), one of its great advantages is its strong insensitivity to the adopted reddening corrections, and to the metallicity of the Cepheid (Storm et al. 2004). With excellent observational data at hand, individual Cepheid distances and radii can be determined with an accuracy of the order of 5% if the adopted  $K$ -band surface brightness-color relation is correct.

A first calibration of this relation coming directly from interferometrically determined angular diameters of Cepheid variables was presented by Nordgren et al. (2002) (hereafter N02). They found a satisfactory agreement with the FG97 calibration, within the combined  $1\sigma$  uncertainties of both surface brightness-color calibrations. Considering more closely the results from N02, an even better agreement is found between the  $F_V(V - K)$  relations *before* the zero point is forced between the different colors. Before this operation, N02 found the relation:

$$F_{V_0} = 3.956 \pm 0.011 - 0.134 \pm 0.005(V - K)_0 \quad (1)$$

that translates, after forcing the zero point to the average of the three selected colors, to the relation:

$$F_{V_0} = 3.941 \pm 0.004 - 0.125 \pm 0.005(V - K)_0. \quad (2)$$

On the other hand, FG97 obtain:

$$F_{V_0} = 3.947 \pm 0.003 - 0.131 \pm 0.003(V - K)_0 \quad (3)$$

From this comparison, it appears that the slope initially determined by N02 for  $F_V(V - K)$  is significantly different both from their final value and from the FG97 relation. This difference could cause a bias due to the averaging of the multi-color zero points. Though small in absolute value, such a bias is of particular importance for  $\ell$  Car, due to its relatively large  $(V - K)$  color.

Another argument in favor of the FG97 surface brightness relation is that it relies on a sample of

11 Cepheids with periods of 4 to 39 days, while the relations established by N02 were derived from the observations of only 3 Cepheids with periods of 5 to 10 days. Such short period Cepheids are significantly hotter than  $\ell$  Car ( $P = 35.5$  days), and a local difference of the slope of the IRSB relations cannot be excluded. For these two reasons, we choose to retain the FG97 calibration for our analysis of the Cepheid  $\ell$  Car in the following section.

## 4. Diameter and distance

### 4.1. Angular diameter

We have combined the photometric data from Pel (1976) and Bersier (2002) to construct the  $V$ -band light curve for  $\ell$  Car. The two data sets are spanning almost 30 years and allow an improved determination of the period of this variable. We find  $P = 35.54804$  days. The time of maximum  $V$  light has been adopted from Szabados (1989) who give a value of  $T_0 = 2440736.230$  which is also in good agreement with the more recent data. The resulting light curve is shown in Fig.1. The  $K$  band light curve is based on the data from Laney & Stobie (1992) and is also shown in Fig.1. The  $(V - K)$  color curve which is needed by the IRSB method has been constructed on the basis of the observed  $V$  band data and a Fourier fit to the  $K$ -band data as described in Storm et al. (2004).

For the radial-velocity curve we have used the data from Taylor et al. (1997) and Bersier (2002). Using the new ephemeris from above we detected a slight offset of  $1.5 \text{ km s}^{-1}$  between the two datasets. We choose to shift the Taylor et al. (1997) dataset by  $-1.5 \text{ km s}^{-1}$  to bring all the data on the well established CORAVEL system of Bersier (2002). We note that the exact radial velocity zero point is irrelevant as the method makes use of *relative* velocities. The combined radial velocity data are displayed in Fig.1.

The application of the IRSB method has followed the procedure described in Storm et al. (2004). We have adopted the same reddening law with  $R_V = 3.30$  and  $R_K = 0.30$ , a reddening of  $E(B - V) = 0.17$  (Ferne 1990), and a projection factor,  $p$ , from radial to pulsational velocity of  $p = 1.39 - 0.03 \log P = 1.343$  (Hindsley & Bell. 1986; Gieren et al. 1993). As discussed by Storm et al. (2004) we only consider the points in the

phase interval from 0.0 to 0.8 (phase zero is defined by the  $V$  band maximum light). We have applied a small phase shift of  $-0.025$  to the radial velocity data to bring the photometric and radial velocity based angular diameters into agreement. We note that a similar phase shift can be achieved by lowering the systemic velocity by  $1.5 \text{ km s}^{-1}$ .

The angular diameter curve obtained from the photometry has been plotted in Fig.2, together with the linear displacement curve. The photometric and interferometric diameter curves are directly compared in Fig. 3, where they are plotted as a function of phase. With these data we can compute the average angular diameters obtained from each technique. For the IRSB, we find an average limb darkened angular diameter  $\overline{\theta_{LD}} = 2.974 \pm 0.046$  milliarcsecond, and for the interferometric measurements we find  $\overline{\theta_{LD}} = 2.992 \pm 0.012$  milliarcsecond. The agreement between these two values is strikingly good. This is a serious indication that the calibration of the surface brightness-color relation (FG97), based on non-pulsating giant stars, does apply to Cepheids.

## 4.2. Distance

The surface brightness method yields a distance of  $560 \pm 6$  pc, and a mean radius of  $R = 179 \pm 2 R_{\odot}$ . The corresponding mean absolute  $V$  magnitude is  $M_V = -5.57 \pm 0.02$  mag and the distance modulus is  $(m - M)_0 = 8.74 \pm 0.05$ . The error estimates are all intrinsic  $1\sigma$  random errors. In addition to these random errors, a systematic error of the order of 4% should be taken into account, as discussed by Gieren et al. (1997). The final IRSB values are thus  $d = 560 \pm 23$  pc, and  $R = 179 \pm 7 R_{\odot}$ . Compared to Storm et al. (2004) we find a significantly (0.24 mag) shorter distance modulus for  $\ell$  Car. This can be explained by the use in the present Letter of the new and superior radial velocity data from Taylor et al. (1997) and Bersier (2002).

K04 found  $d = 603^{+24}_{-19}$  pc, using the interferometric angular diameters and a subset of the radial velocity data used here. To make the comparison more relevant, we determined the distance and radius using the same data (interferometric diameters from Table 1 and radial velocity from Taylor et al. (1997) and Bersier (2002) – see above), the same ephemeris, and the same projection factor (see Sect 4.1). Using the method of K04, we

find a distance  $d = 566^{+24}_{-19}$ , and a linear radius  $R = 182^{+8}_{-7} R_{\odot}$ . This is in excellent agreement with the values obtained from the IRSB method.

This 6% difference in the distances based on interferometric diameters (603 pc for K04 versus 566 pc here) has two major causes. First, the  $p$ -factor used in the present paper is  $\sim 1.3\%$  smaller than in K04. The choice of the reference used for the  $p$ -factor has currently an impact of a few percents on its value. This indicates that the average value of the  $p$ -factor for a given Cepheid is currently uncertain by at least a similar amount, and this systematic error translates linearly to the distance determination.

Secondly, the use of a different – and superior – data set for the radial velocity makes the radius curve different from K04. In particular the amplitude is smaller here than in K04 by  $\sim 3\%$ . This is likely due to the more complete phase coverage that we have here, and possibly also to a different choice of spectral lines to estimate the radial velocity. This amplitude difference translates linearly on the distance through the BW method.

## 5. Conclusion

The main point of our paper is to show that with a consistent treatment of the data, the internal accuracy of both methods (IRSB or interferometry) is extremely good: the angular diameter variation observed using the VLTI agrees very well with that derived from the  $F_V(V - K)$  version of the IRSB technique as calibrated by FG97. For all the interferometric measurements, the corresponding IRSB angular diameter at the same phase lies within the combined  $1\sigma$  error bars of the two measurements (Fig. 3). Even more importantly, the mean angular diameter of the Cepheid as derived from both independent sets of angular diameter determination are in excellent agreement, within a few percents.

Unfortunately, this is not equivalent to say that the Cepheid distance scale is calibrated to a 1% accuracy. We have drawn attention to remaining sources of systematic errors that can affect Cepheid radii and distances up to several percents. As an illustration of these sources, K04 obtain a distance  $d = 603^{+24}_{-19}$  pc for  $\ell$  Car, while we obtain  $d = 566^{+24}_{-19}$  pc from the same interferometric data.

We have already shown that most of the 6% dif-

ference (equivalent to  $1.3\sigma$ ) can be explained by the use of different radial velocity data and projection factor. Another thing to consider is the phase interval used. K04 used measurements over the whole pulsation cycle whereas in the IRSB technique, one avoids the phase interval 0.8–1 (Fig. 2). During that phase interval, that corresponds to the rebound of the atmosphere around the minimum radius, energetic shock waves are created. As discussed by Sabbey et al. (1995), they produce asymmetric line profiles in the Cepheid spectrum. Recent modeling using a self-consistent dynamical approach also show that the  $\tau = 1$  photosphere may not be comoving with the atmosphere of the Cepheid during its pulsation, at the 1 % level (Nardetto et al. in preparation). Such an effect would impact the  $p$ -factor, modify the shape of the radial velocity curve, and thus bias the amplitude of the radius variation, possibly up to a level of a few percents. As the BW method (either its classical or interferometric versions) relies linearly on this amplitude, a bias at this level presently cannot be excluded.

The interferometric BW method is currently limited to distances of 1–2 kpc, due to the limited length of the available baselines. The IRSB technique on the other hand can reach extra-galactic Cepheids as already demonstrated by Gieren et al. (2000, for the LMC) and by Storm et al. (2004, for the SMC). Using high precision interferometric measurements of  $\ell$  Car and other Cepheids, it will be possible to calibrate the IRSB method down to the level of a few percents. From the present comparison, we already see that this fundamental calibration will be very similar to the calibration found by FG97 and Nordgren et al. (2002).

WPG acknowledges support for this work from the Chilean FONDAP Center for Astrophysics 15010003.

## REFERENCES

- Bersier, D. 2002, *ApJS*, 140, 465
- Claret, A. 2000, *A&A*, 363, 1081
- Fernie, J.D. 1990, *ApJS*, 72, 153
- Fouqué, P., & Gieren, W.P. 1997, *A&A*, 320, 799 (FG97)
- Gieren, W.P., Fouqué, P., & Gómez, M. 1997, *ApJ*, 488, 74 (GFG97)
- Gieren, W.P., Fouqué, P., & Gómez, M. 1998, *ApJ*, 496, 17
- Gieren, W.P., Barnes, T.G., & Moffett, T.J. 1993, *ApJ*, 418, 135
- Gieren, W.P., Storm, J., Fouqué, P., Mennickent, R. E., & Gómez, M. 2000, *ApJ*, 533, L107
- Glindemann, A., Abuter, R., Carbognani, F., et al. 2000, *Proc. SPIE*, 4006, 2
- Hindsley, R. B., & Bell, R. A. 1986, *PASP*, 98, 881
- Kervella, P., Coudé du Foresto, V., Glindemann, A., & Hofmann, R. 2000, *Proc. SPIE*, 4006, 31
- Kervella, P., Gitton, Ph., Ségransan, D., et al. 2003, *Proc. SPIE*, 4838, 858
- Kervella, P., Nardetto, N., Bersier, D., et al. 2004, *A&A*, in press, astro-ph/0311525 (K04)
- Lane, B. F., Kuchner, M. J., Boden, A. F., Creech-Eakman, M. , Kulkarni, S. R. 2000, *Nature*, 407, 485
- Lane, B. F., Creech-Eakman, M. J., & Nordgren, T. E. 2002, *ApJ*, 573, 330
- Laney, C. D., & Stobie, R.S 1992, *A&A Suppl. Ser.*, 93, 93
- Marengo, M., Karovska, M., Sasselov, D. D., et al. 2003, *ApJ*, 589, 968
- Marengo, M., Karovska, M., Sasselov, D. D., & Sanchez, M. 2004, *ApJ*, 603, in press, astro-ph/0312349
- Nardetto, N., et al. 2004, in preparation
- Nordgren, T. E., Lane, B. F., Hindsley, R. B., & Kervella, P. 2002, *ApJ*, 123, 3380 (N02)

Pel, J. W. 1976, A&A Suppl. Ser., 24, 413

Petrov, R., Malbet, F., Richichi, A., et al. 2000, Proc. SPIE, 4006, 68

Sabbey, C.N., Sasselov, D.D., Fieldus, M.S., et al. 1995, ApJ, 446, 250

Sasselov, D. D. & Karovska M. 1994, ApJ, 432, 367

Storm, J., Carney, B.W., Gieren, W.P., Fouqué, P., Latham, D.W., & Fry, A.M. 2004, A&A, in press, astro-ph/0401211

Szabados, L. 1989, Comm. Konkoly Observatory, 94, 1

Taylor, M.M., Albrow, M.D., Booth, A.J., & Cottrell, P.L. 1997, MNRAS, 292, 662

Welch, D.L. 1994, AJ, 108, 1421

Table 1: Angular diameter measurements of  $\ell$  Car. The statistical and systematic calibration uncertainties are mentioned separately in brackets.

Julian Date	Phase	$\theta_{LD}$ (mas)
2452453.498	0.618	$3.054 \pm 0.113$ [0.041,0.105]
2452739.564	0.665	$2.891 \pm 0.087$ [0.076,0.043]
2452740.569	0.693	$2.989 \pm 0.047$ [0.018,0.044]
2452741.717	0.726	$2.993 \pm 0.039$ [0.026,0.029]
2452742.712	0.754	$2.899 \pm 0.056$ [0.035,0.043]
2452743.698	0.781	$2.758 \pm 0.076$ [0.074,0.016]
2452744.634	0.808	$2.794 \pm 0.035$ [0.032,0.013]
2452745.629	0.836	$2.675 \pm 0.098$ [0.097,0.017]
2452746.620	0.864	$2.775 \pm 0.046$ [0.023,0.040]
2452747.599	0.891	$2.699 \pm 0.129$ [0.127,0.026]
2452749.576	0.947	$2.645 \pm 0.078$ [0.077,0.012]
2452751.579	0.003	$2.753 \pm 0.033$ [0.028,0.017]
2452755.617	0.117	$2.970 \pm 0.113$ [0.113,0.013]
2452763.555	0.340	$3.194 \pm 0.034$ [0.009,0.033]
2452765.555	0.396	$3.212 \pm 0.034$ [0.011,0.033]
2452766.550	0.424	$3.210 \pm 0.035$ [0.011,0.033]
2452768.566	0.481	$3.188 \pm 0.037$ [0.011,0.035]
2452769.575	0.509	$3.189 \pm 0.022$ [0.018,0.012]
2452770.535	0.536	$3.160 \pm 0.022$ [0.020,0.009]
2452771.528	0.564	$3.136 \pm 0.020$ [0.017,0.010]
2452786.620	0.989	$2.727 \pm 0.064$ [0.012,0.063]

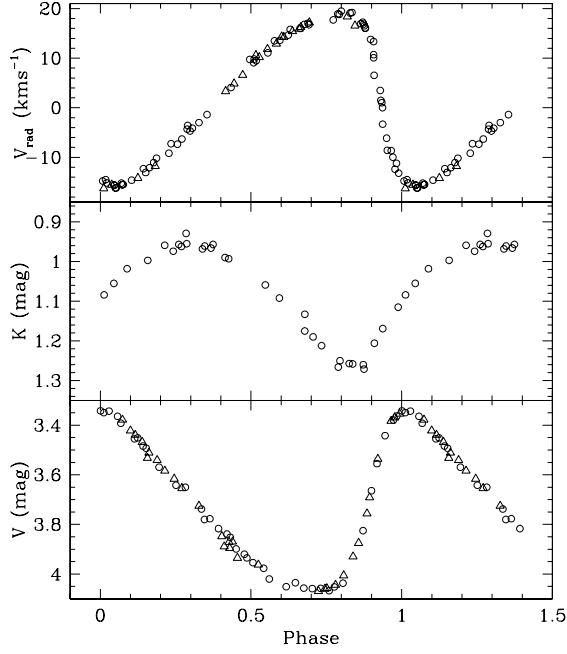


Fig. 1.— Radial velocity curve of  $\ell$  Car (upper panel) using data from Taylor et al. (1997) shifted by  $-1.5 \text{ km s}^{-1}$  (circles) and from Bersier (2002) (triangles). The  $K$  band photometric measurements (middle panel) were taken from Laney & Stobie (1992). We have relied on Pel (1976) (circles) and Bersier (2002) (triangles) for the  $V$  band data.

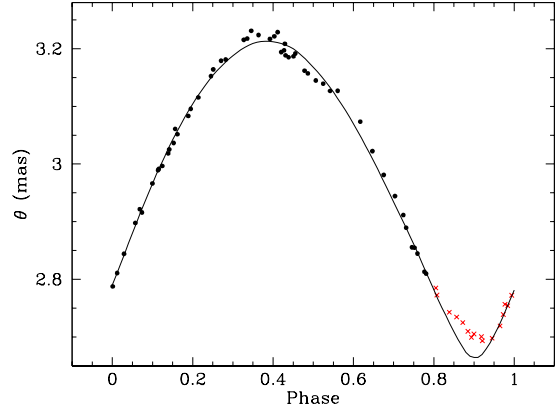


Fig. 2.— Photometric angular diameters plotted against phase for our best fitting distance. The solid curve represents the integrated radial velocity curve of  $\ell$  Car for the adopted distance.

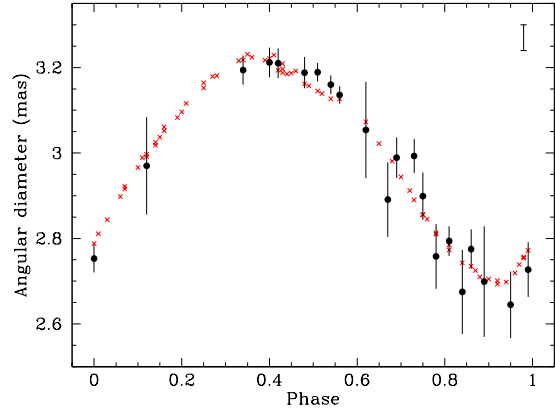


Fig. 3.— The interferometrically determined angular diameters, plotted against phase (solid dots) with the angular diameters derived with the infrared surface brightness method overplotted (crosses). In the upper right corner a typical error bar for the surface brightness method data is shown.